Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 20 June 2023

Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced

PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 6 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 7 2 8 2 0 A 0 1 2 0

Pearson

1. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of $3.2\,\text{m}\,\text{s}^{-2}$

Find

(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

(a) Use suvat

s

$u = 0$ "at rest"

$v = v$

$a = 3.2\,\text{ms}^{-2}$

$t = 5\,\text{s}$

Use Formula:

$v = u + at$

$v = 0 + 3.2(5)$

$v = 16\,\text{ms}^{-1}$ B1

(b) Use suvat again

$s = s$

$u = 0$

~~v~~

$a = 3.2\,\text{ms}^{-2}$

$t = 5\,\text{s}$

Use formula

$s = ut + \frac{1}{2}at^2$

$s = (0)(5) + \frac{1}{2}(3.2)(5)^2$ M1

$s = 40\,\text{m}$ A1

P 7 2 8 2 0 A 0 2 2 0

Question 1 continued

(Total for Question 1 is 3 marks)

2.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

friction!

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\text{m}\,\text{s}^{-2}$

(b) Find the value of F

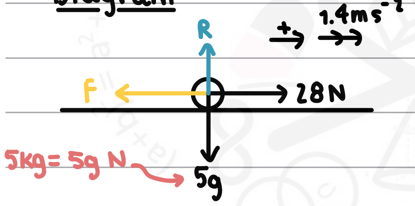
(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

Diagram



(a) Since it is not moving vertically, use $\Sigma F_y = 0$

$R = 5g = 49\text{ N}$ B1

(b) Since it's accelerating, use $\Sigma F_x = ma$

$28 - F = (5)(1.4)$ M1

$F = 21\text{ N}$ A1

(c) We know $F = \mu R$ as it's moving

$21 = \mu \times 5g$

$\mu = \frac{21}{5 \times 9.8}$

$\mu = \frac{3}{7}$

$\mu = 0.43$ to 2sf (μ has no units!)

B1 ! The exercise asks for 2sf!

P 7 2 8 2 0 A 0 4 2 0

Question 2 continued

(Total for Question 2 is 4 marks)

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$ **(3)**

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$ **(2)**

(c) the acceleration of P at time t seconds **(2)**

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$ **(2)**

(a) Substitute $t = 0$ into the equation for velocity

$v = (0^2 - 3(0) + 7)i + (2(0)^2 - 3)j$

$v = 7i - 3j$ B1

Use Pythagoras' Theorem to get $|v|$, which is the speed

$|v| = \sqrt{7^2 + (-3)^2}$ M1

$= \sqrt{58}\ \mathrm{ms^{-1}}$ A1

(b) Get the ratio between the i and j components of v and equate to $\frac{i}{j}$

$\frac{t^2 - 3t + 7}{2t^2 - 3} = \frac{1}{1}$ M1

$t^2 - 3t + 7 = 2t^2 - 3$ solve for t

$0 = t^2 + 3t - 10$

$0 = (t+5)(t-2)$

~~$t = -5$~~ $t = 2$ A1

Reject as $t > 0$.

(c) To get acceleration from velocity, we need to differentiate

$v = (t^2 - 3t + 7)i + (2t^2 - 3)j$

$a = \frac{dv}{dt} = (2t - 3)i + (4t)j$ ☆ simple differentiation: $\frac{dy}{dx} = nx^{n-1}$

M1A1

(d) "perpendicular to i" ∴ a parallel to j, so i component = 0

$2t - 3 = 0$ M1

$t = \frac{3}{2} = 1.5\,s$ A1

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 9 marks)

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P passes through the point A.

At time $t = 5\,\text{s}$, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})\,\text{m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})\,\text{m}$.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

(a) Use suvat A → B

s
$u = (-16i - 3j)\,ms^{-1}$
$v = v$
$a = (2.4i + j)\,ms^{-2}$
$t = 5$

Use Formula:

$v = u + at$

$v = \begin{pmatrix}-16\\-3\end{pmatrix} + \begin{pmatrix}2.4\\1\end{pmatrix} \times 5$ M1

$v = \begin{pmatrix}-16\\-3\end{pmatrix} + \begin{pmatrix}12\\5\end{pmatrix} = \begin{pmatrix}-4\\2\end{pmatrix}$ Velocity A1

Use Pythagoras' Theorem to get $|v|$, which is speed.

$|v| = \sqrt{(-4)^2 + 2^2}$ M1

$= \sqrt{20} = 2\sqrt{5}\,ms^{-1}$ A1

Use suvat A → C

$s_0 = (44i - 10j)\,m$
$s = (4i + cj)\,m$
$u = (-16i - 3j)\,ms^{-1}$
v
$a = (2.4i + j)\,ms^{-2}$
$t = T$

Use Formula

$s = s_0 + ut + \frac{1}{2}at^2$

$\begin{pmatrix}4\\c\end{pmatrix} = \begin{pmatrix}44\\-10\end{pmatrix} + \begin{pmatrix}-16\\-3\end{pmatrix}T + \frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}T^2$ M1

(b) Use i components to get T:

A1 $0 = 40 - 16T + 1.2T^2$ factorize

$0 = (3T - 10)(0.4T - 4)$

$T = \frac{10}{3}$ $T = 10$

$T > 5 \therefore T = 10\,s$ A1

P 7 2 8 2 0 A 0 8 2 0

Question 4 continued

(c) Use the j components to get c.

$c = -10 - 3T + \frac{1}{2}T^2$ M1M1

Substitute T=10

$c = -10 - 3(10) + \frac{1}{2}(10)^2$

$c = -40 + \frac{1}{2}(100)$

$c = 10$ A1

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5.

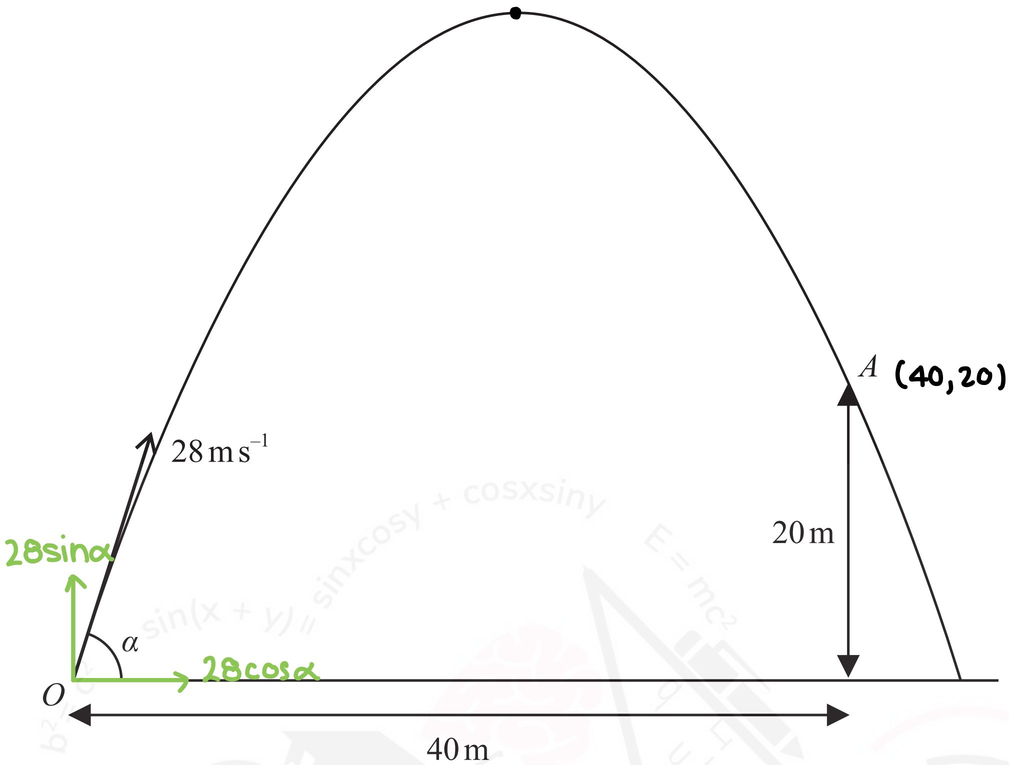


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$ **(2)**

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$ **(5)**

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A. **(3)**

The model does not include air resistance.

(d) State one other limitation of the model. **(1)**

Question 5 continued

(a) Horizontally – use $s = ut$

$s = 40$, $u = 28\cos\alpha$, $t = T$

$40 = 28\cos\alpha T$ M1

$T = \frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha} \rightarrow T = \frac{10}{7\cos\alpha}$ hence shown A1 Eq1

(b) Vertically – use suvat

↑+ $s = 40$, $u = 28\sin\alpha$, ~~v~~, $a = -g$, $t = T$

Use Formula $s = ut + \frac{1}{2}at^2$

$40 = 28T\sin\alpha + \frac{1}{2}(-g)T^2$

$40 = 28T\sin\alpha - 4.9T^2$ Eq. 2

M1 A1

Substitute in $T = \frac{10}{7\cos\alpha}$:

$40 = 28\sin\alpha \times \frac{10}{7\cos\alpha} - 4.9\left(\frac{10}{7\cos\alpha}\right)^2$ M1

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$40 = 40 \times \frac{\sin\alpha}{\cos\alpha} - 4.9\left(\frac{100}{49\cos^2\alpha}\right)$

$\sec^2\alpha = 1 + \tan^2\alpha$

$40 = 40\tan\alpha - 10\sec^2\alpha$

$40 = 40\tan\alpha - 10(1 + \tan^2\alpha)$ M1

$4 = 4\tan\alpha - 1 - \tan^2\alpha$

A1 $0 = \tan^2\alpha - 4\tan\alpha + 3$ hence shown

(c) Solve $0 = \tan^2\alpha - 4\tan\alpha + 3 \rightarrow 0 = (\tan\alpha - 3)(\tan\alpha - 1)$

M1 $\tan^{-1}(3) = \alpha°$ $\tan\alpha = 3$ $\tan\alpha = 1$

At the highest point, the vertical velocity is instantaneously 0.

Use suvat:

↑+ $s = H$, $u = 28\sin\alpha$, $v = 0$, $a = -g$

Use Formula $v^2 = u^2 + 2as$

$0^2 = (28\sin(\tan^{-1}(3)))^2 + 2(-g)(H)$ M1

$2gH = (28\sin(\tan^{-1}(3)))^2$

$H = \frac{(28\sin(\tan^{-1}(3)))^2}{2g}$

A1 $H = 36\text{m}$ max. height

(d) The spin of the ball and wind effect are ignored B1

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6.

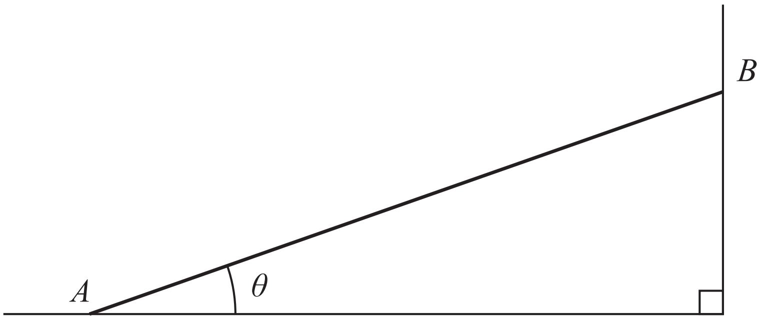


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium. $\therefore F = \mu R$

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

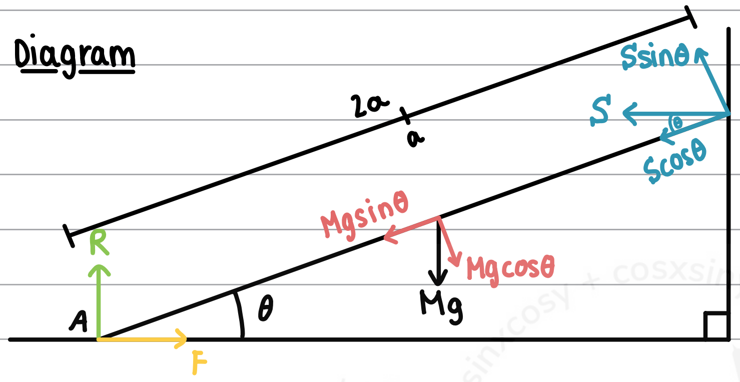
(1)

DO NOT WRITE IN THIS AREA

Question 6 continued

(a) The frictional force is acting to the right as the reaction force at B acts to the left and friction opposes it. Also, due to the weight that makes the rod tend towards slipping left and friction prevents this. B1

Diagram



★ We take moments about A because this way, the perpendicular distance to R and F is 0. So we don't need to consider R and F, making our moments equation much simpler.

(b) To get S we will use moments about point A. M1

Since it's at rest, $\Sigma M_A = 0$

$\Sigma M_A = a \times Mg\cos\theta - 2a \times S\sin\theta = 0$ A1

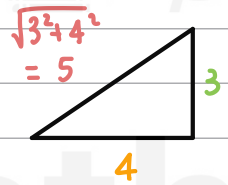
$a Mg\cos\theta = 2a S\sin\theta$ cancel a's

$\frac{1}{2} Mg\cos\theta = S\sin\theta$

$\frac{1}{2} Mg \frac{\cos\theta}{\sin\theta} = S$ Identity: $\frac{\cos\theta}{\sin\theta} = \cot\theta$

$S = \frac{1}{2} Mg\cot\theta$ hence shown A1

$\tan\alpha = \frac{3}{4}$



$\therefore \cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

(c) We can resolve both vertically and horizontally.

The rod is at rest, $\therefore \Sigma F_y = 0$ and $\Sigma F_x = 0$.

$\Sigma F_y = 0$: B1 $R = Mg$

$\Sigma F_x = 0$: B1 $F = S = \frac{1}{2} Mg\cot\theta$ (from (a))

We are told it's in limiting equilibrium $\therefore F = \mu R$.

Substitute into $F = \mu R$: B1

$S = \mu R$

$\frac{1}{2} Mg\cot\theta = \mu R$ $\cot\theta = \frac{1}{\tan\theta}$

$\frac{1}{2} Mg \frac{1}{\frac{3}{4}} = \mu \times Mg$ cancel Mg dM1

$\frac{1}{2} \times \frac{4}{3} = \mu \longrightarrow \mu = \frac{2}{3}$ value of μ A1

P 7 2 8 2 0 A 0 1 7 2 0

Turn over

Question 6 continued

(d) At Point A we have the forces R and F.

Using Pythagoras' Theorem to get the magnitude of the resultant.

$$|\text{Resultant}| = \sqrt{F^2 + R^2} \quad \text{M1}$$

$F = \frac{2}{3}Mg$, $R = Mg$] from (c)

$$|\text{Res}| = \sqrt{\left(\tfrac{2}{3}Mg\right)^2 + (Mg)^2} \quad \text{M1}$$

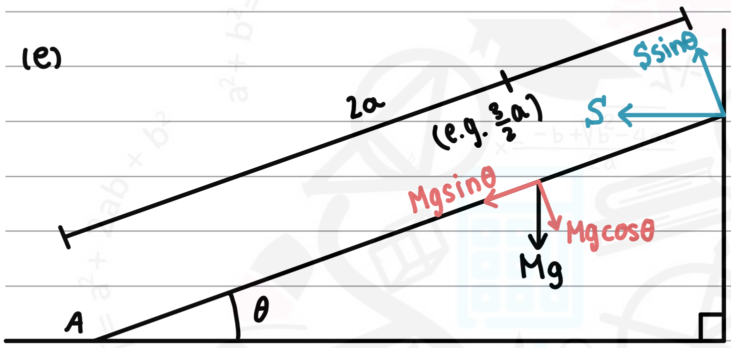
$$= \sqrt{\tfrac{4}{9}M^2g^2 + M^2g^2}$$

$$= \sqrt{\tfrac{13}{9}M^2g^2}$$

$$= \sqrt{\tfrac{13}{9}}\,Mg = \frac{\sqrt{13}}{3}Mg$$

magnitude of resultant A1

(e)



If the weight acts closer to B, the moment about A due to the weight will be larger as the perpendicular distance is larger. This means S will be larger as well to "balance out" the larger moment of the weight, so that the rod remains at rest. B1

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

TOTAL FOR MECHANICS IS 50 MARKS